



Estimating Photometric Redshifts using Gaussian Process Regression in the Sloan Digital Sky Survey (and 2MASS)

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<http://astrophysics.arc.nasa.gov/~mway/Stanford-200807.pdf>

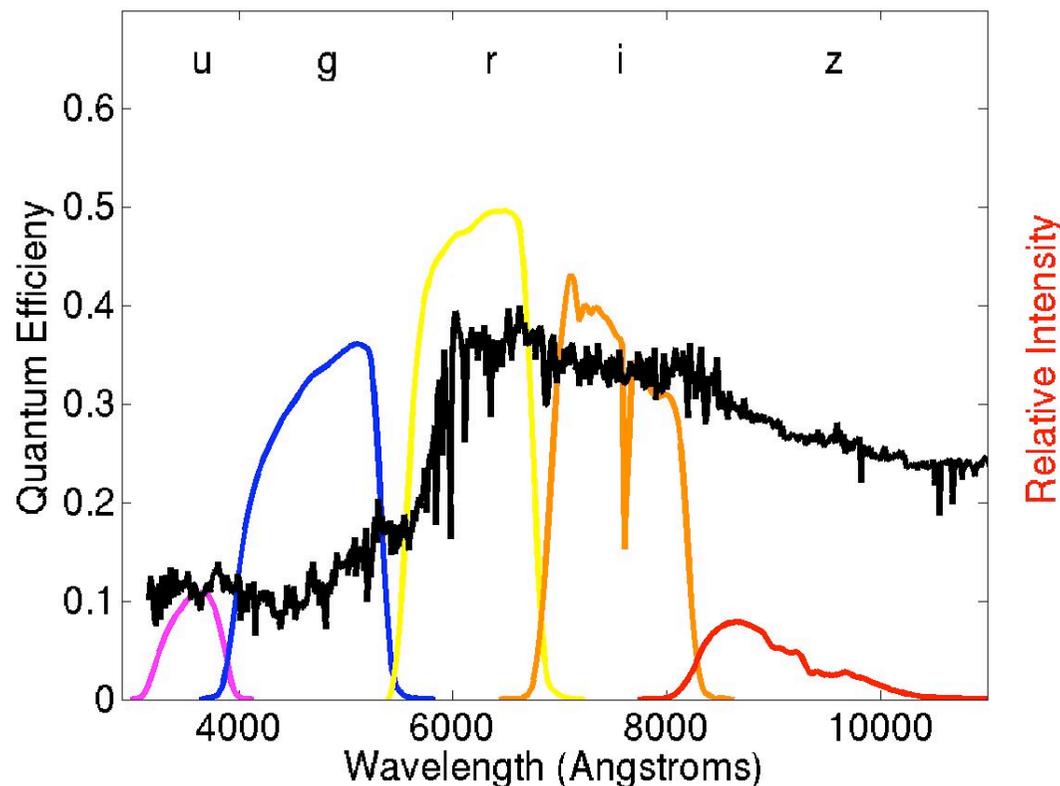
- What are Photometric Redshifts?
- A quick background on common training set methods
- What is Gaussian Process Regression?
- Do different kinds of Kernels matter?
- Does better quality photometry matter?
- How many galaxies do I need to get a good fit?
- Do SDSS morphological indicators help?

What are Photometric Redshifts?

Photometric Redshifts: A **rough** estimate of the redshift of a galaxy without having to measure a spectrum.

$$Z_{\text{spec}} = (\lambda_{\text{measured}} - \lambda_{\text{rest}}) / \lambda_{\text{rest}}$$

$$Z_{\text{photo}} = z(\mathbf{C}, \mathbf{m})$$

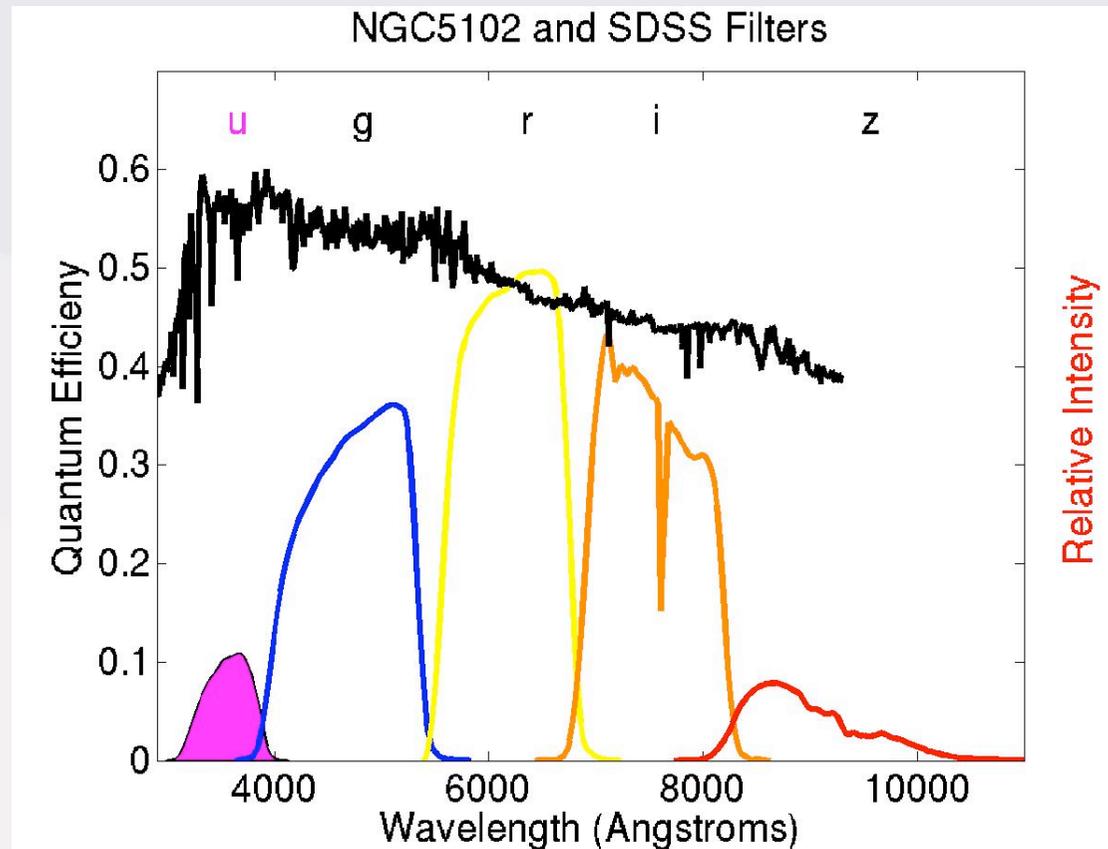


What are Photometric Redshifts?

$$Z_{\text{spec}} = (\lambda_{\text{measured}} - \lambda_{\text{rest}}) / \lambda_{\text{rest}}$$

$z=0.0$

$$z_{\text{photo}} = z(C, m)$$



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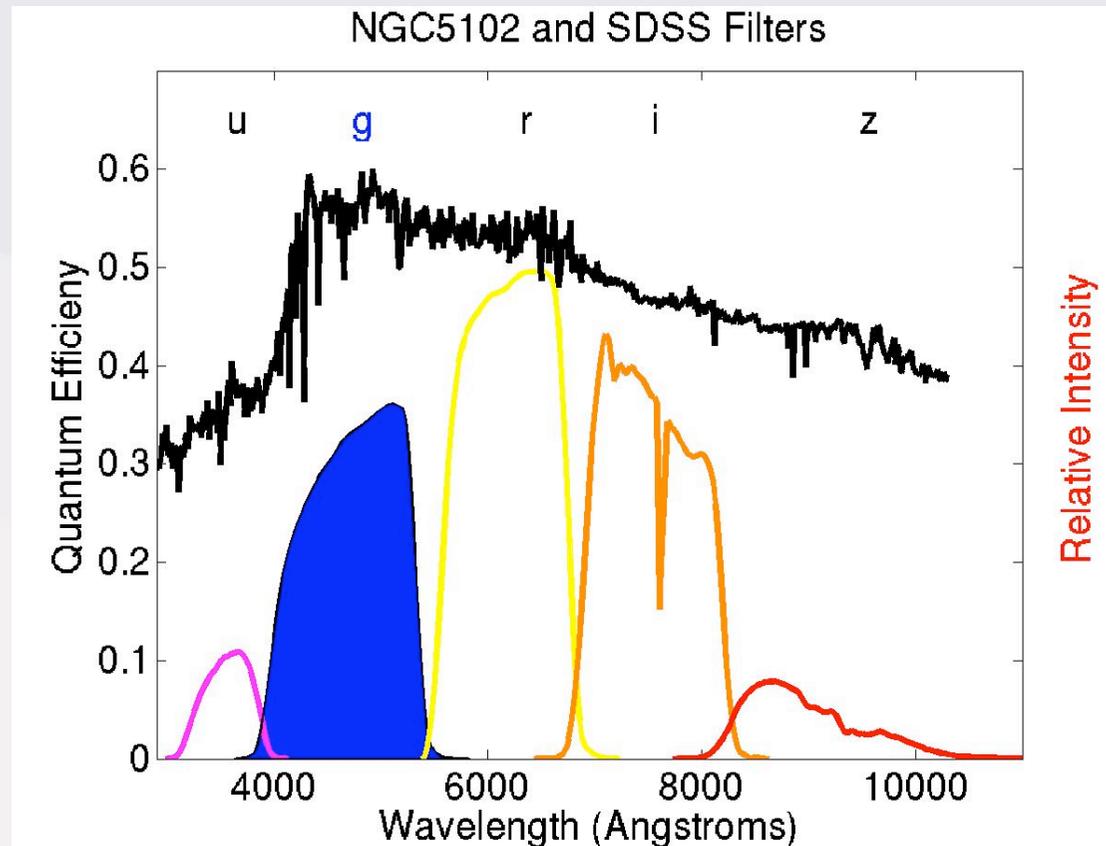


What are Photometric Redshifts?

$$Z_{\text{spec}} = (\lambda_{\text{measured}} - \lambda_{\text{rest}}) / \lambda_{\text{rest}}$$

$$z_{\text{photo}} = z(C, m)$$

$z \sim 0.06$ (18000 km/s)



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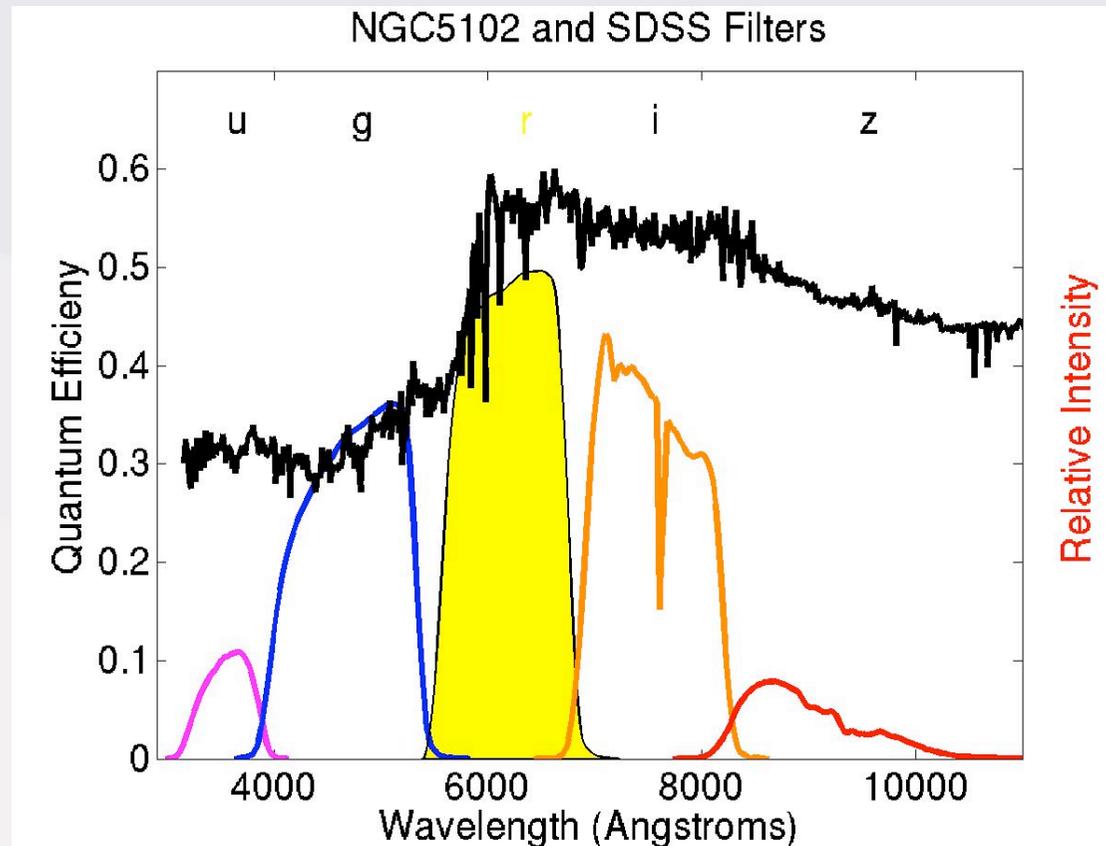


What are Photometric Redshifts?

$$Z_{\text{spec}} = (\lambda_{\text{measured}} - \lambda_{\text{rest}}) / \lambda_{\text{rest}}$$

$z \sim 0.6$

$$Z_{\text{photo}} = z(C, m)$$



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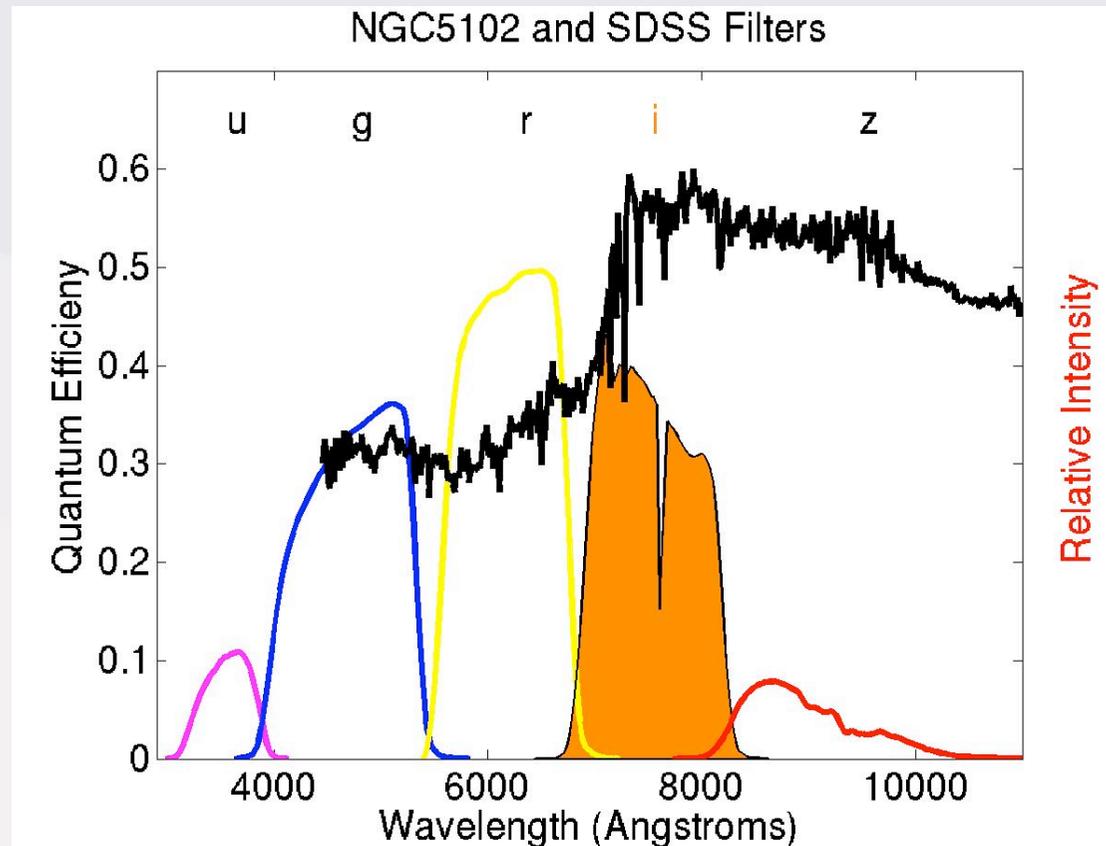


What are Photometric Redshifts?

$$Z_{\text{spec}} = (\lambda_{\text{measured}} - \lambda_{\text{rest}}) / \lambda_{\text{rest}}$$

$$z_{\text{photo}} = z(C, m)$$

$z \sim 0.90$



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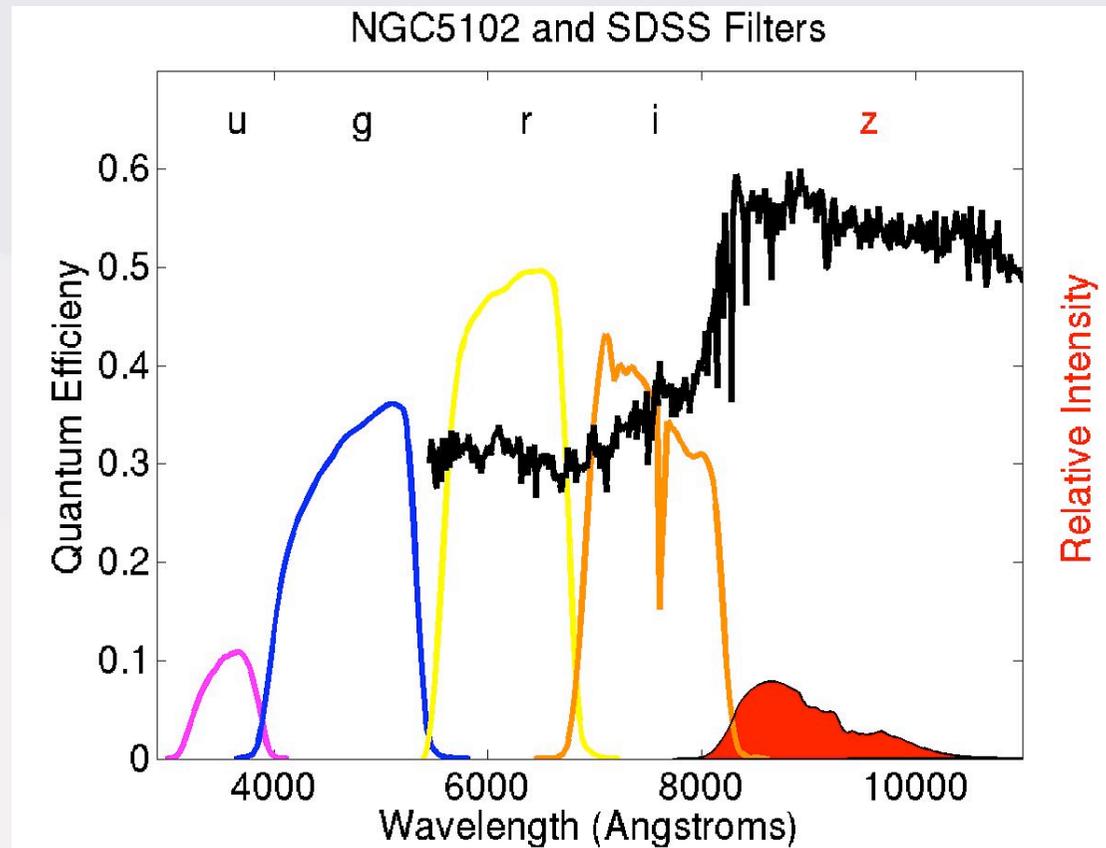


What are Photometric Redshifts?

$$Z_{\text{spec}} = (\lambda_{\text{measured}} - \lambda_{\text{rest}}) / \lambda_{\text{rest}}$$

$$z_{\text{photo}} = z(C, m)$$

$z \sim 1.10$



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Photo-z methods

1.) Spectral Energy Distribution (SED) Fitting:

- model based approach
- uses redshifts derived from spectra of artificial galaxies (e.g. Bruzual & Charlot)

2.) Training-Set methods:

- **empirical approach**
- **uses *spectroscopic* redshifts from a sub-sample of galaxies with the same band-pass filters**



Photo-z The Empirical Approach

Training Set Methods need a sub-sample of Galaxies:

- of known spectroscopic redshift
- with a comparable range of **magnitudes** (u g r i z) to our Photometric survey objects
- These will be our “Training Samples”





“Training Set” Methods

Galaxy Photometric Redshift Prediction History

- Linear Regression was first tried in the 1960s
- Quadratic & Cubic Regression (1970s)
- Polynomial Regression (1980s)
- Neural Networks (1990s)
- Kd Trees & Bayesian Classification Approaches (1990s)
- Support Vector Machines & GP Regression (2000s)

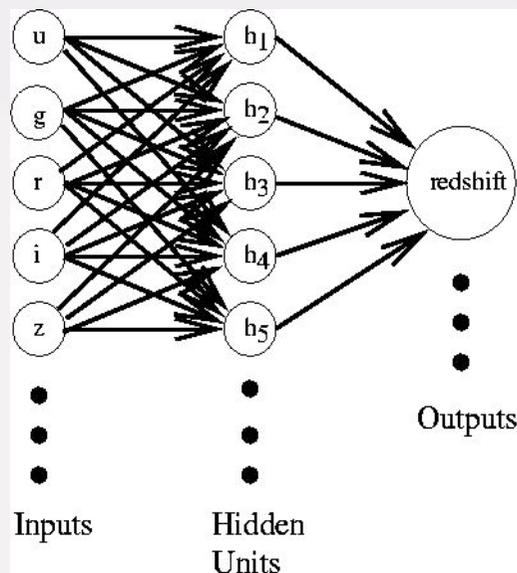


Gaussian Process Regression fitting

Gaussian Process Regression \Leftrightarrow Kernel Methods

Kernel Methods have replaced Neural Networks in the Machine Learning literature

WHY?: given a large # of hidden units \Rightarrow GP (Neal 1996).



$$h_n > 100$$

$\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow$

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Kernel Methods - Gaussian Process Regression

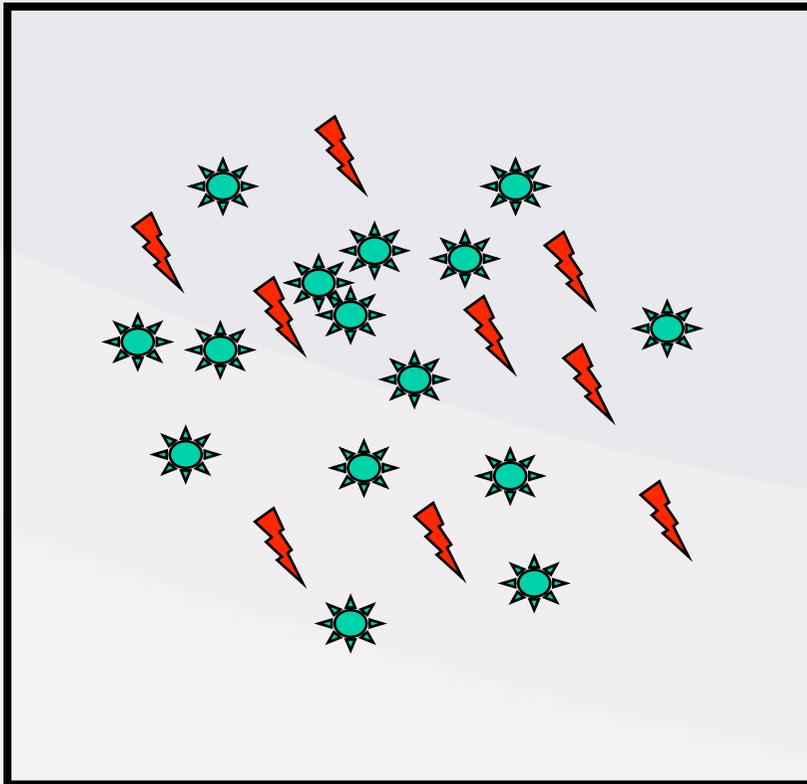
GP regression builds a linear model in a very high dimensional *parameter space* (“feature space” \rightarrow Hilbert space).

- One can map the data using a function $F(x)$ [kernel] into this high (or infinite) dimensional *parameter space* where one can perform linear operations.



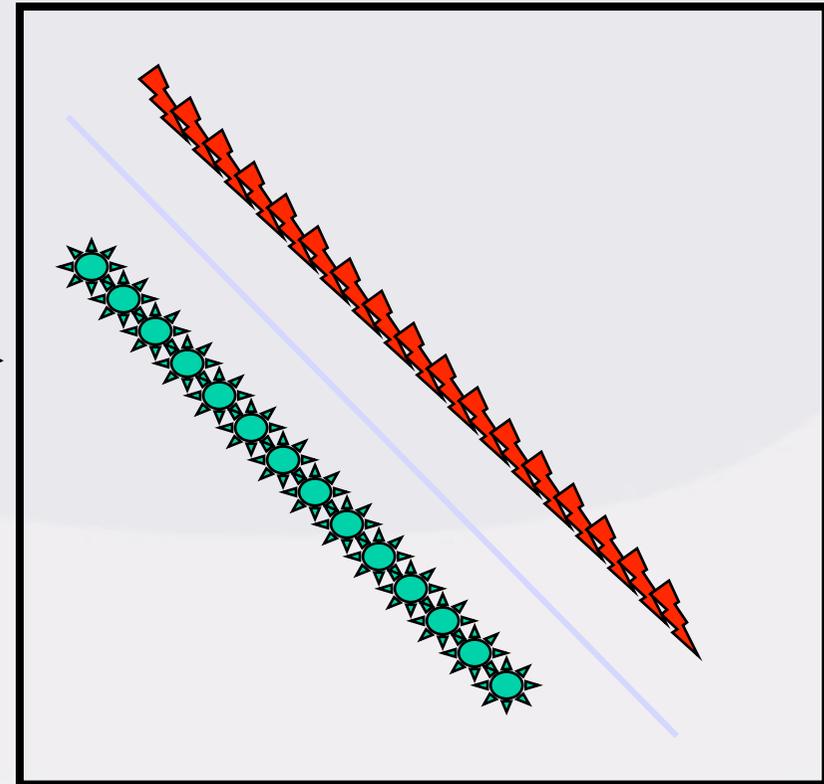
The value of kernels

Original Data without Kernel



Data in original space: highly complex decision boundaries.

Mapped Data using Kernel



Data in high dimensional feature space after mapping through $F(x)$ can yield simple decision boundaries.

GP Regression (Kernels)

GP Advantages:

- Small input data training samples (good for higher redshifts?) yet low errors
- Over fitting is eliminated by use of proper priors
- Realistic estimation of individual redshift errors



GP Disadvantages:

- Possibly large CPU time requirements
 - The Kernel (Covariance Matrix) **can** be large:
 $K = (\lambda^2 I + XX^T)^2$ if $X = 5 \times 180,000$ (our case) then
 K is a matrix $180,000 \times 180,000$ and we have:
$$y^* = K^* (\lambda^2 I + K)^{-1} y$$
 - Need to invert this large K matrix - $O(N^3)$ operation
- Kernel Selection is ambiguous?



GP: Which Kernel??

Using GPs Part I: Pick a transfer/covariance function

Matern Class Fcn

$$k(r) = \frac{2^{l-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu r}}{l} \right)^\nu J_\nu \left(\frac{\sqrt{2\nu r}}{l} \right) \quad \nu \rightarrow \infty$$

Radial Basis Fcn

$$k(r) = \exp\left(-\frac{r^2}{2l^2}\right)$$

Rational Quadratic

$$k_{RQ}(r) = 1 + \left(\frac{r^2}{2\alpha l^2} \right)^{-\alpha}$$

Polynomial

$$k(x, x') = \left(\sigma_o^2 + x^T \sum_p x' \right)^p$$

Neural Nets

$$k_{NN}(x, x') = \frac{2}{\pi} \sin^{-1} \left(\frac{2x^T \Sigma x'}{\sqrt{(1 + 2x^T \Sigma x)(1 + 2x'^T \Sigma x')}} \right)$$





GP Regression How-to

Using GPs Part II: That matrix inversion...

With our SDSS (DR3) Main Galaxy spectroscopic sample (180,000 galaxies) the matrix size is 180,000 x 180,000

- Need a SSI supercomputer with a LOT of ram and cpu time?
- One can take a random sample of ~1000 galaxies & invert that while bootstrapping n times from full sample (Paper I)
- **However, some low-rank matrix approximations work well** (Cholesky Decomposition, Subset of Regressors, Projected Process Approx, etc.)



Results: Other authors

Method Name	σ_{rms}	Dataset ¹	Inputs ²	Source
CWW	0.0666	SDSS-EDR	ugriz	Csabai et al. (2003)
Bruzual-Charlot	0.0552	SDSS-EDR	ugriz	Csabai et al. (2003)
ClassX	0.0340	SDSS-DR2	ugriz	Suchkov et al. (2005)
Polynomial	0.0318	SDSS-EDR	ugriz	Csabai et al. (2003)
Support Vector Machine	0.0270	SDSS-DR2	ugriz	Wadadekar (2005)
Kd-tree	0.0254	SDSS-EDR	ugriz	Csabai et al. (2003)
Support Vector Machine	0.0230	SDSS-DR2	ugriz+r50+r90	Wadadekar (2005)
Artificial Neural Network	0.0229	SDSS-DR1	ugriz	Collister & Lahav (2004)



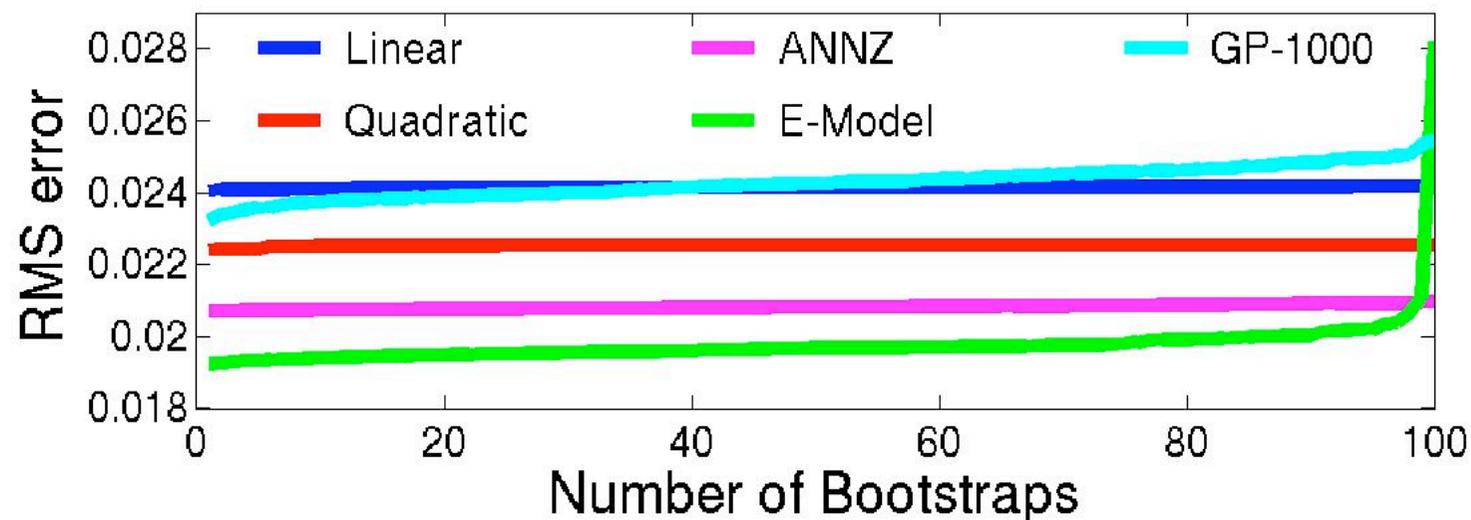
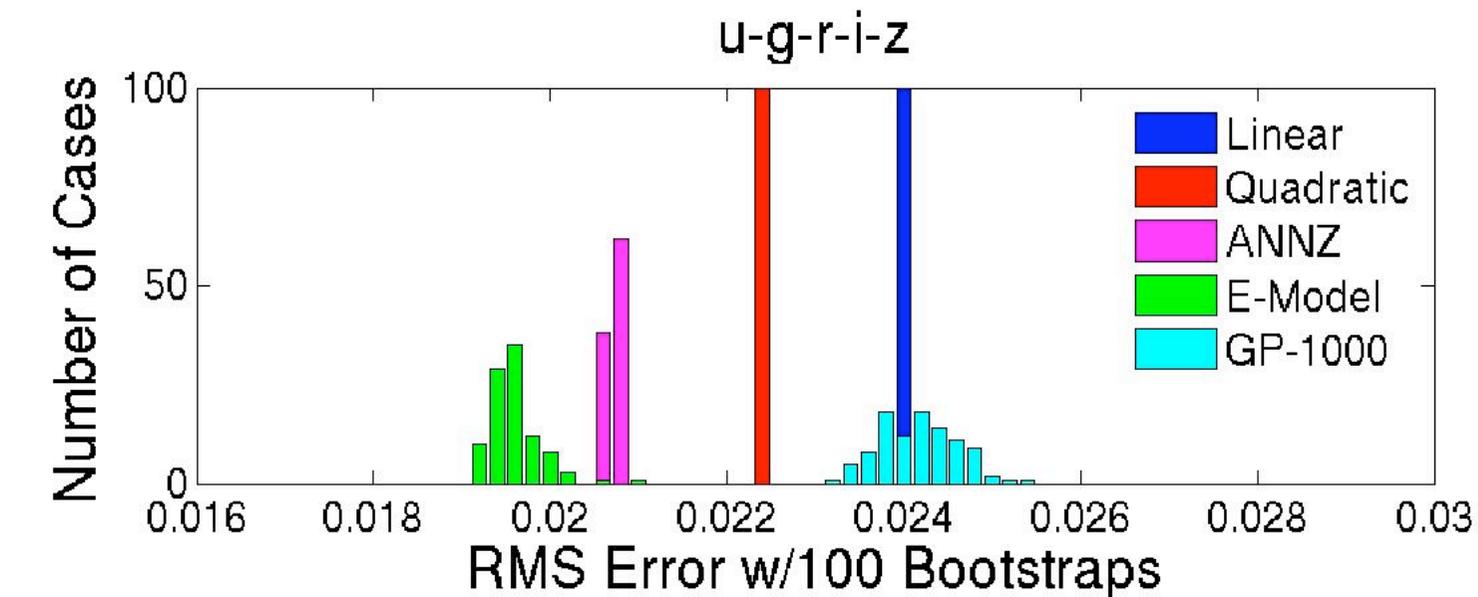
GP Regression (Results)

Results: SDSS (DR3) Main Galaxy Sample

- Paper I: Compared linear, quadratic, Neural Networks and GPs on the SDSS
- With ONLY 1000 samples GPs performed well compared to the other methods
- Paper II: With *low-rank matrix inversion approximations* GPs performed better than all other methods

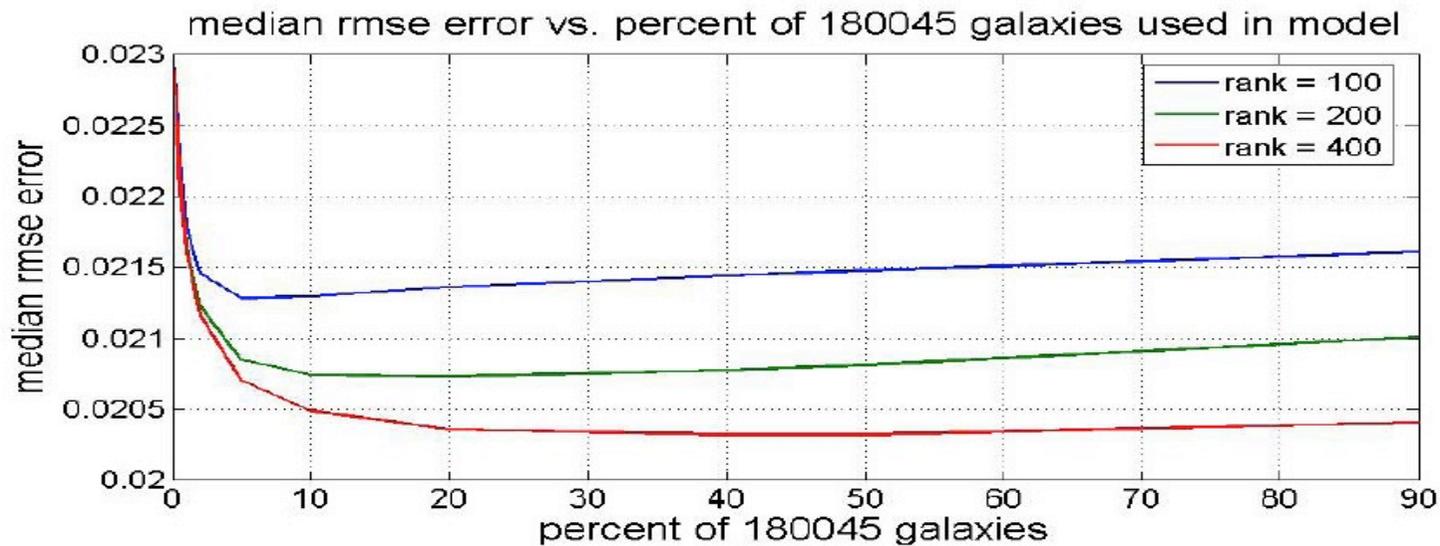
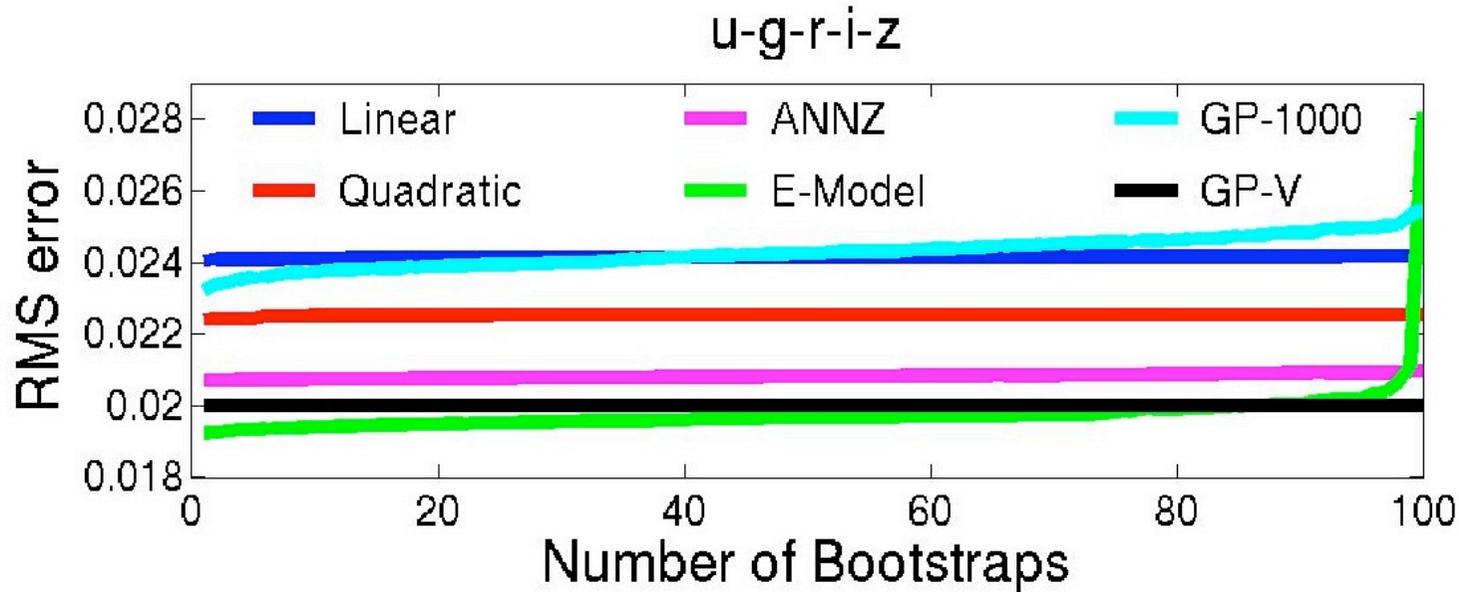


Paper I Results: Comparing Methods



Latest Results: Comparing Methods

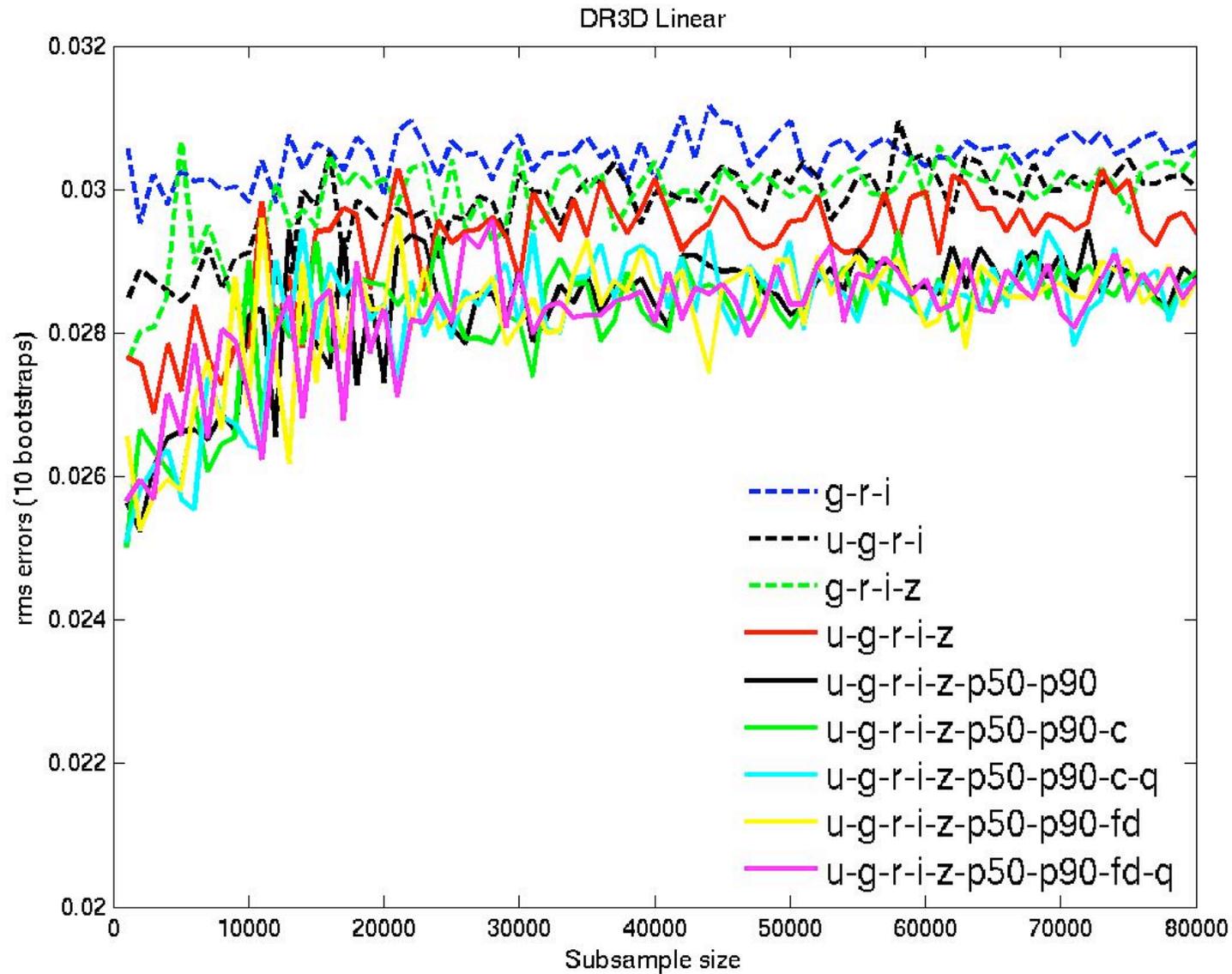
GP-V: Rank=1000 for 36000
↓



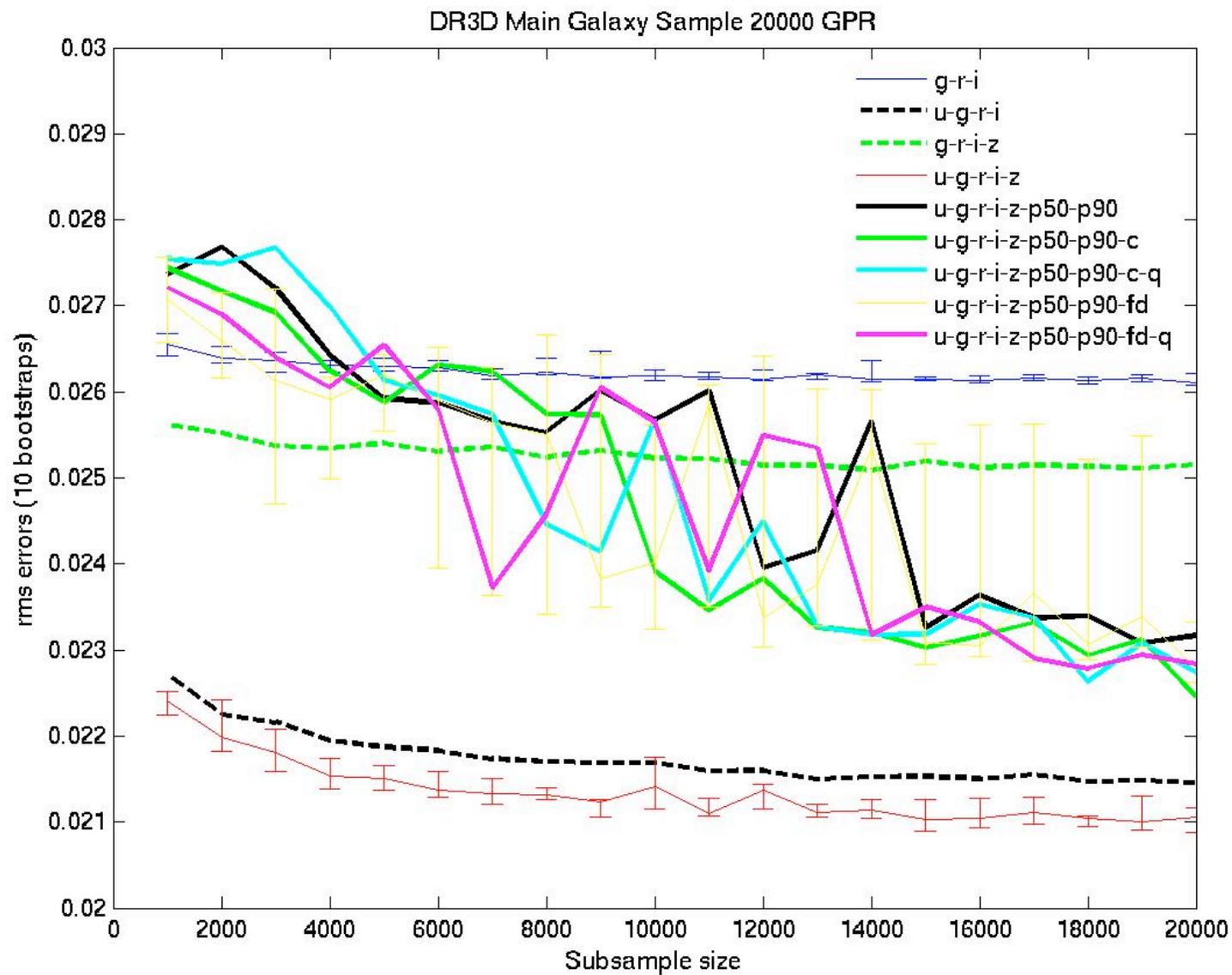
↑ Beyond 20% (~36000) Rank 400 is fairly flat



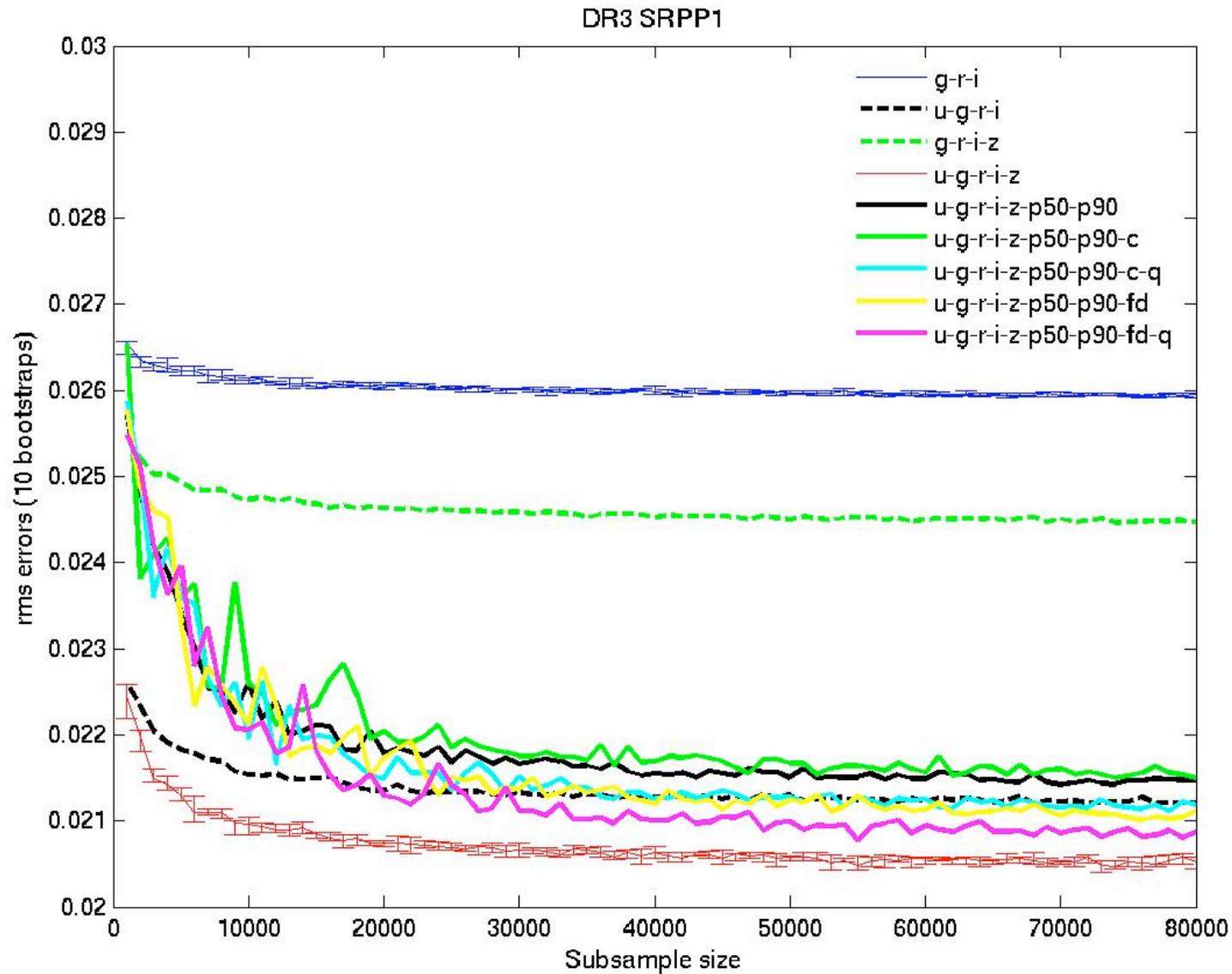
Main Galaxy Sample 80000 Linear



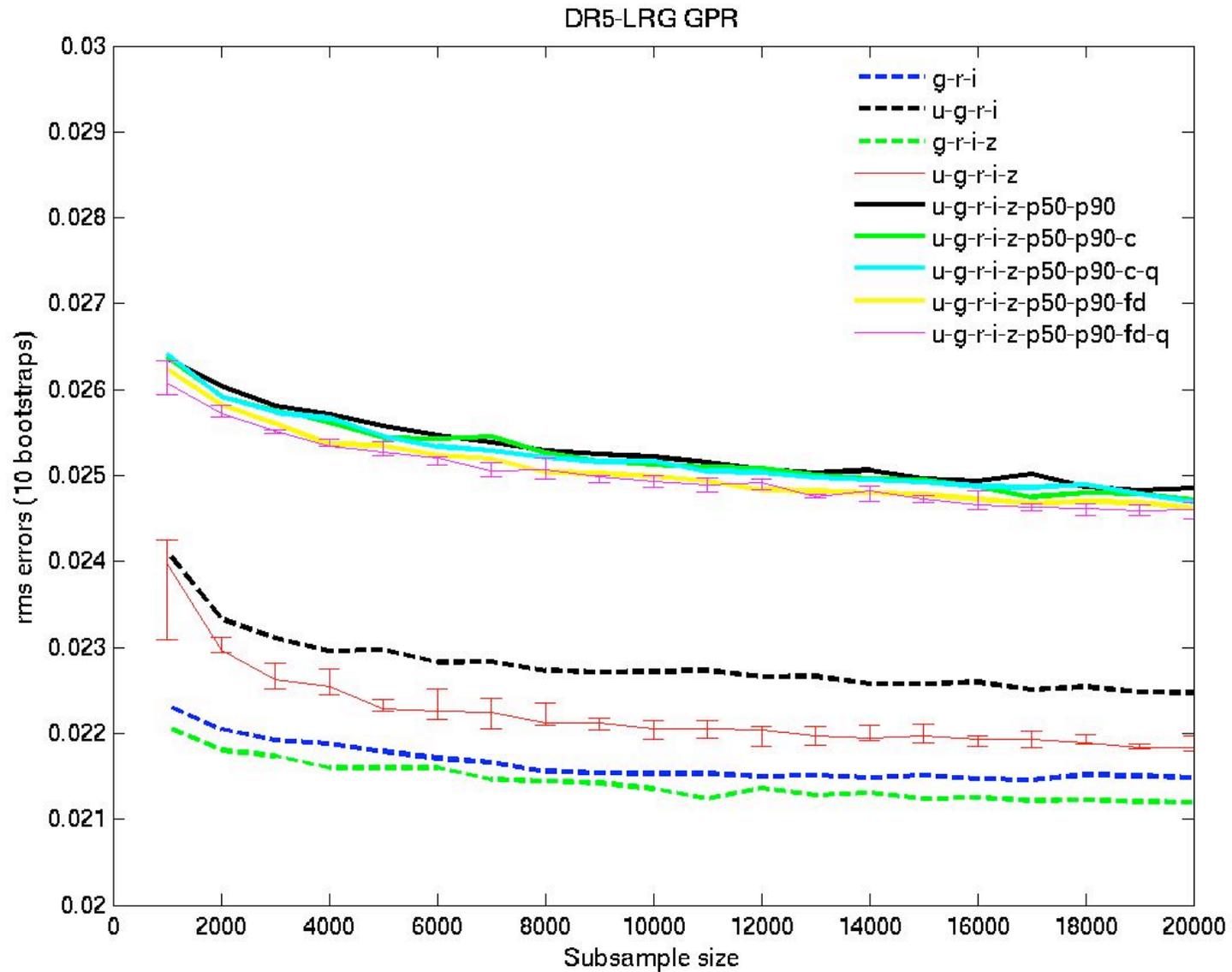
Main Galaxy Sample 20000 GPR



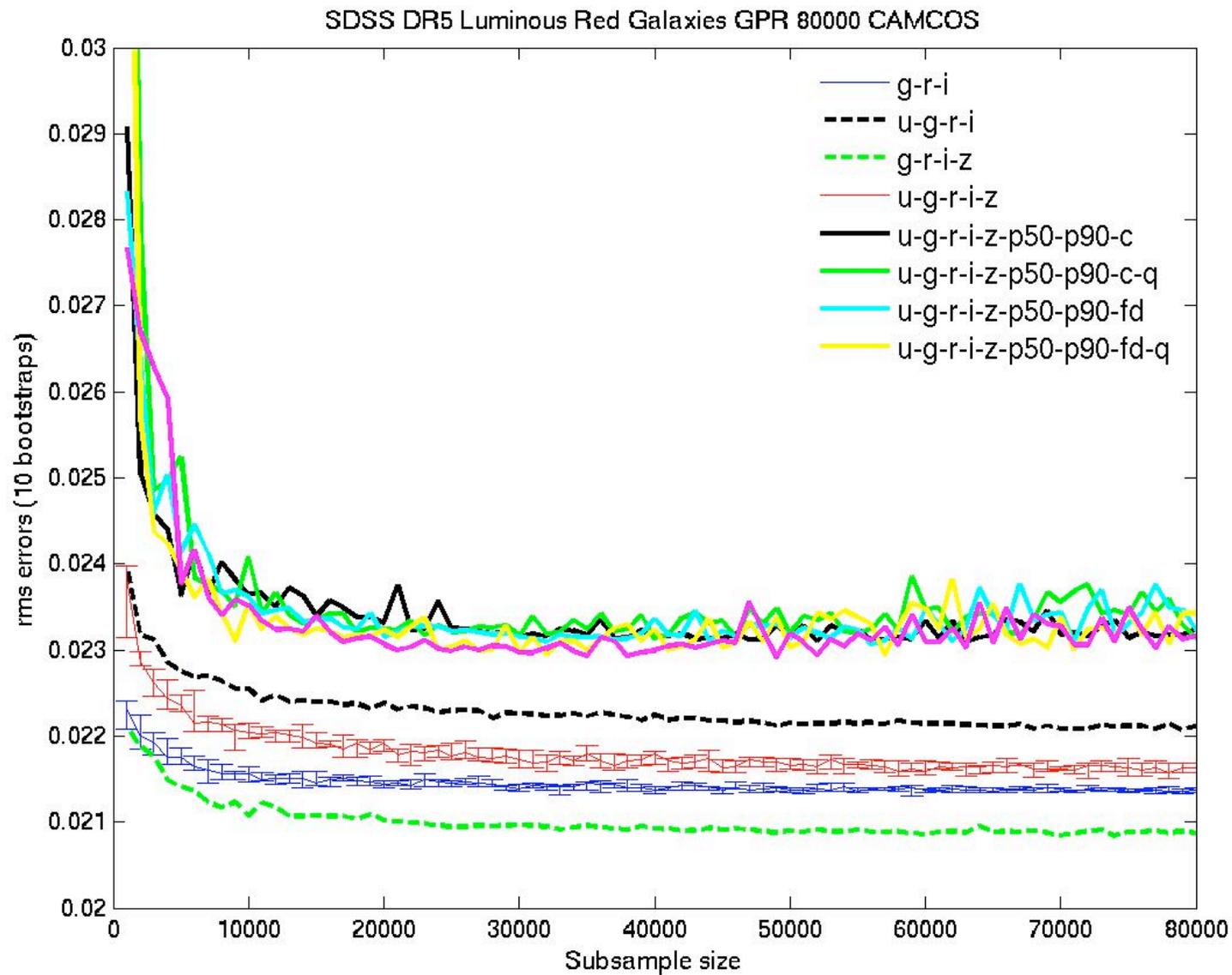
Main Galaxy Sample 80000 GPR



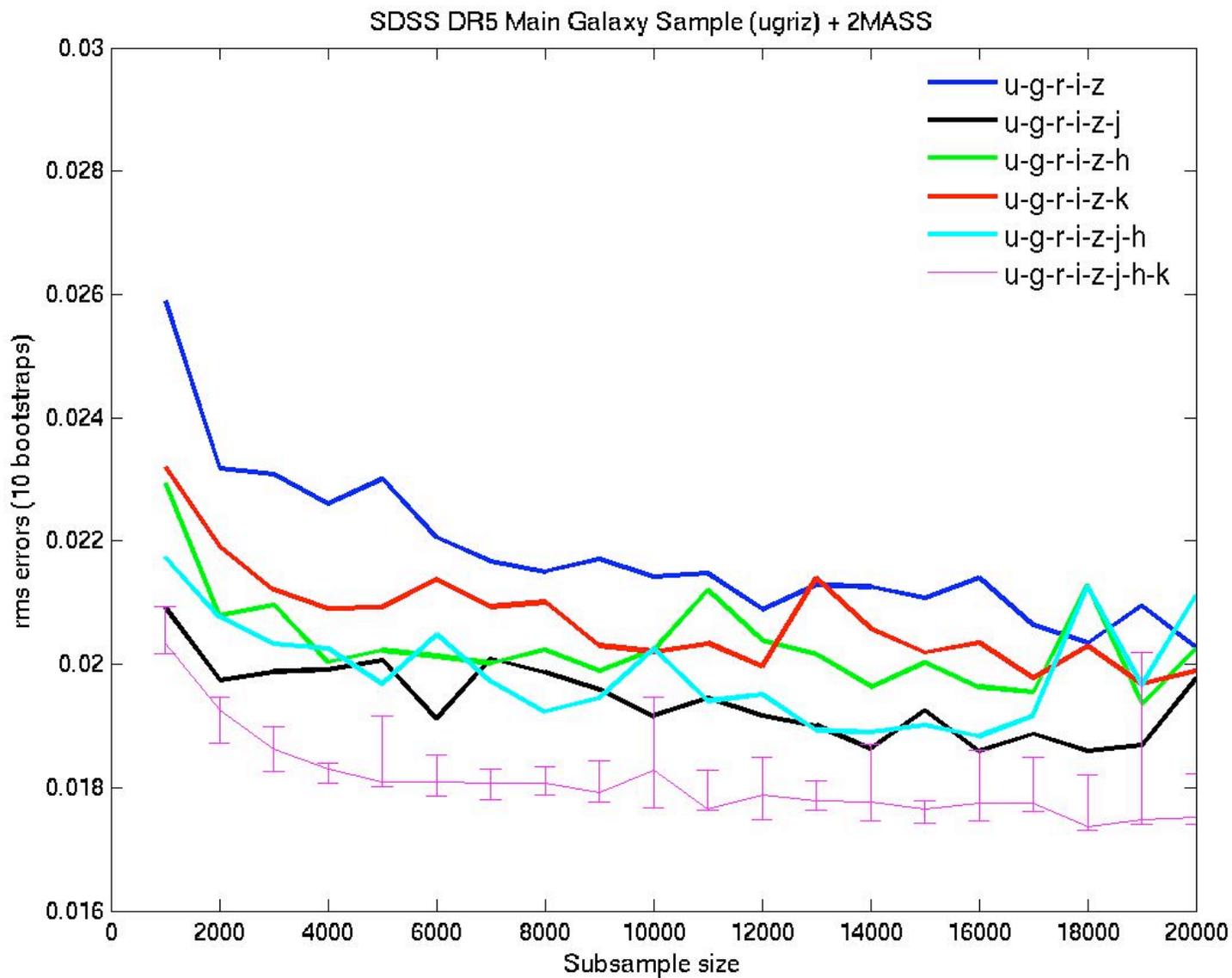
Luminous Red Galaxies 20000 GPR



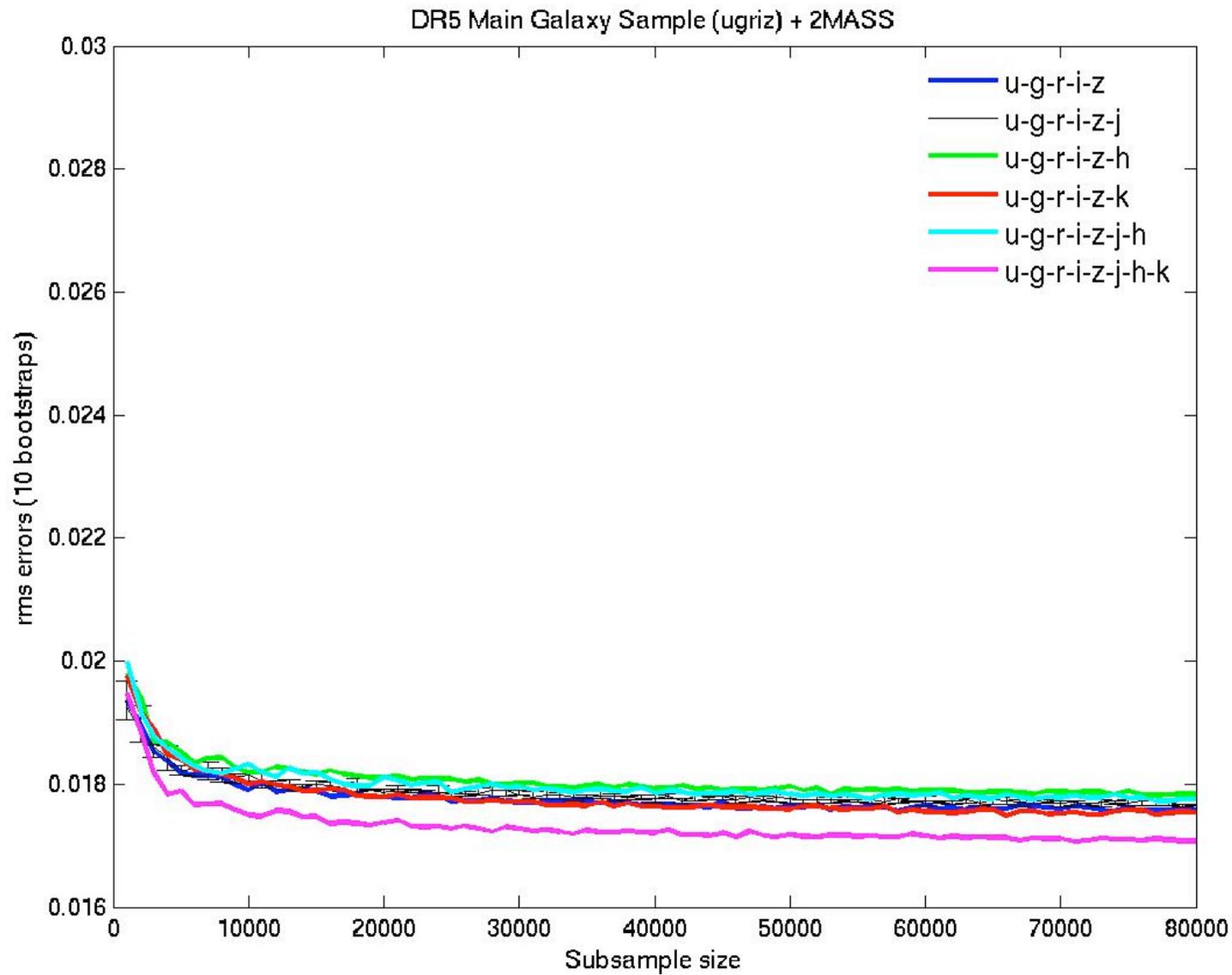
LRG 80000 GPR



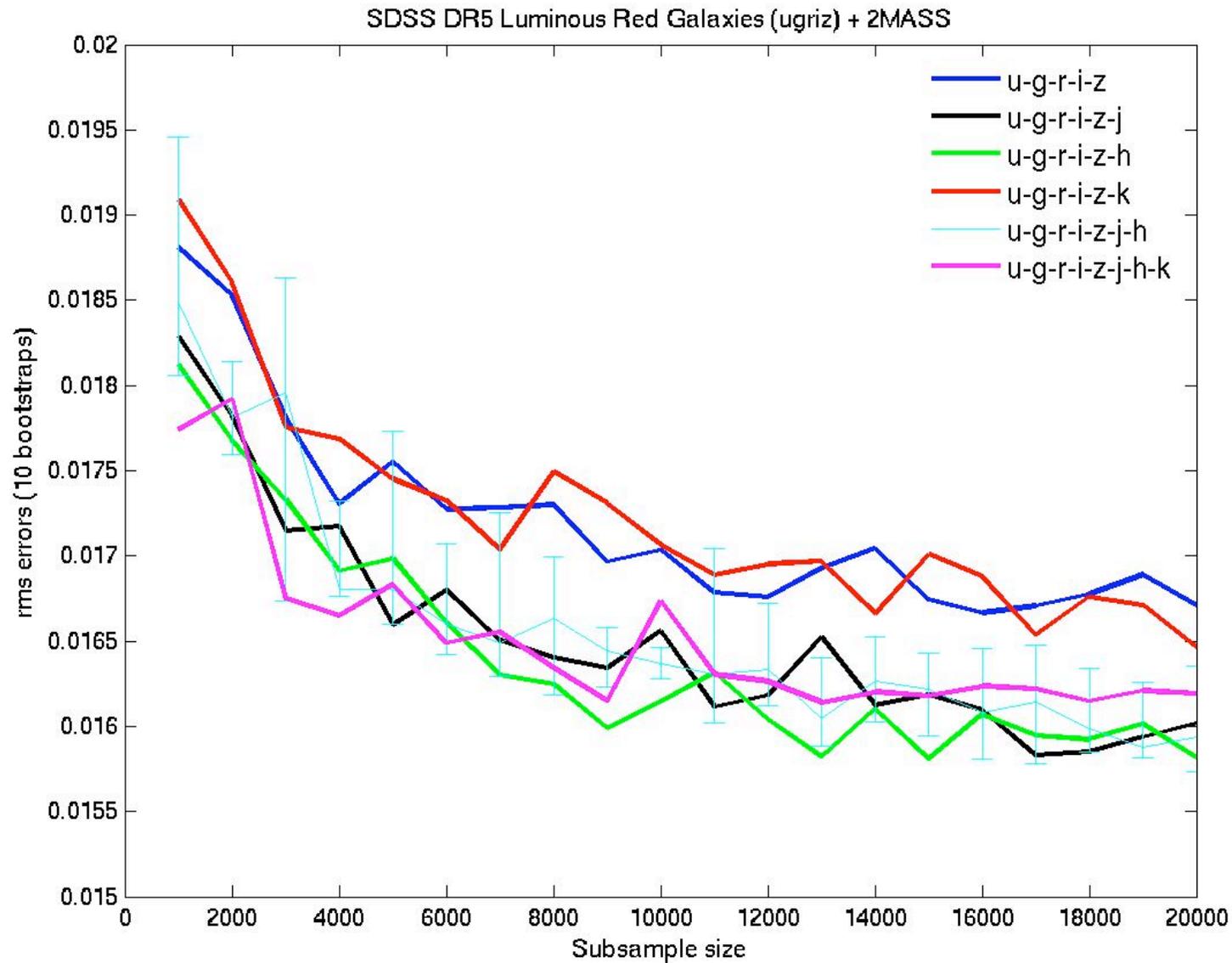
MGS + 2MASS 20000



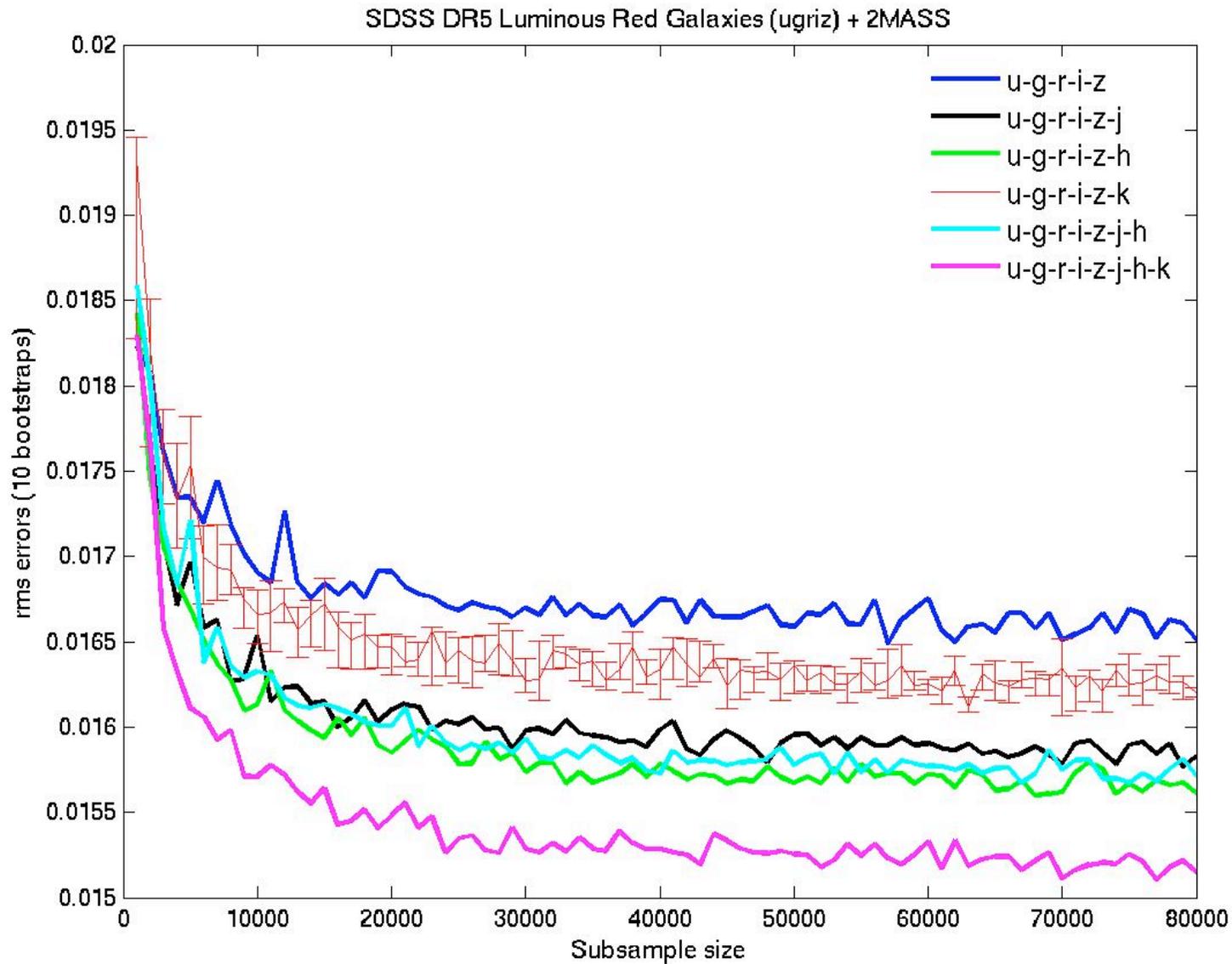
MGS + 2MASS 80000



LRG+ 2MASS 20000



LRG+ 2MASS 80000





Results?

- Morphology does not *generally* increase accuracy of photo-z estimation *with GPR*
- Better quality photometry and/or removal of error outliers does not help
- Additional Near IR filters (2MASS) increase accuracy
- Galaxy classification helps: MGS vs LRG
- Optimal filters? u-g-r-i-z and j-h-k

